



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Lecture 17: Interference based Sensors

Michelson and Mach-Zehnder Interferometers: Sensing Applications

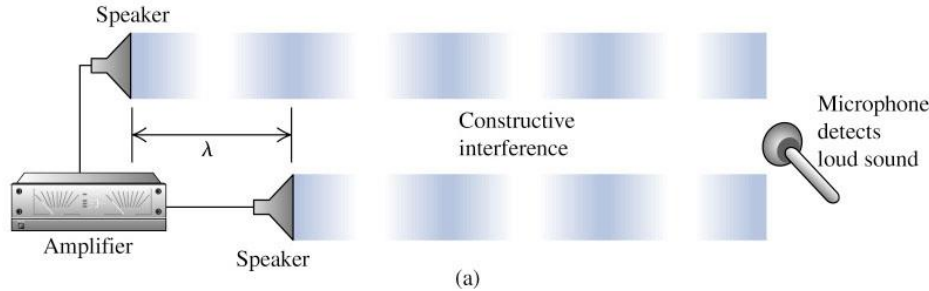
Sachin Kumar Srivastava

Department of Physics



Interference

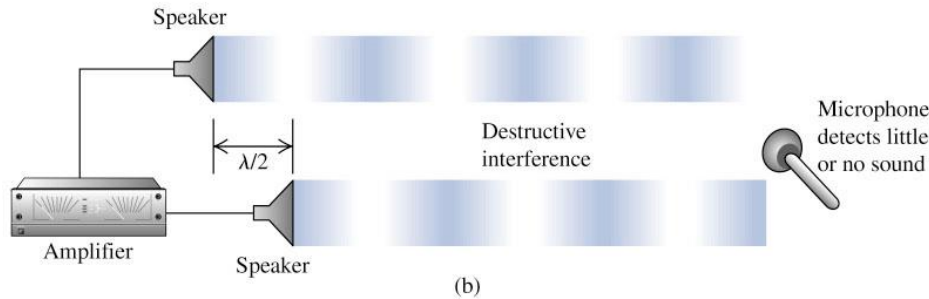
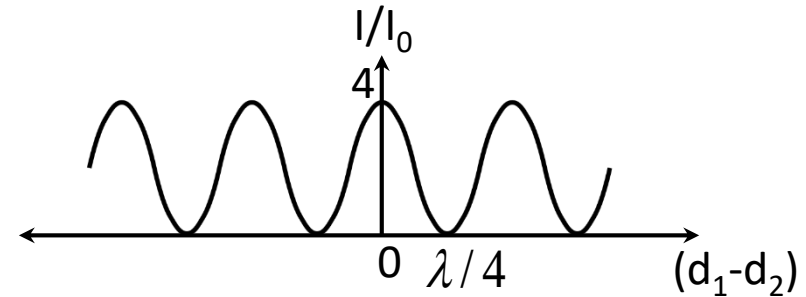
Intensity Profile:



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$I_1 = I_2 = I_0 \Rightarrow I = 2I_0(1 + \cos \delta) = 4I_0 \cos^2 \frac{\delta}{2}$$

$$= 4I_0 \cos^2 \left[\frac{2\pi}{\lambda} \cdot 2(d_1 - d_2) \right]$$

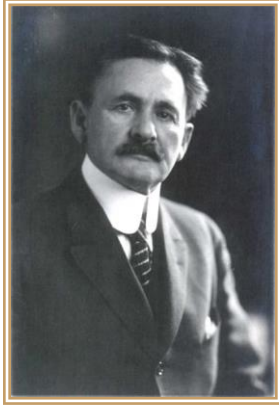


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$\delta = 2m\pi, m = 0, 1, 2, \dots$ *Constructive Interference*

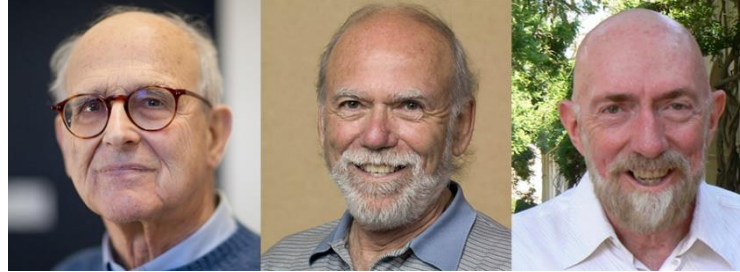
$\delta = (2m + 1)\pi, m = 0, 1, 2, \dots$ *Destructive Interference*

Michelson Interferometer



Albert Abraham Michelson

(Nobel Prize, 1907)

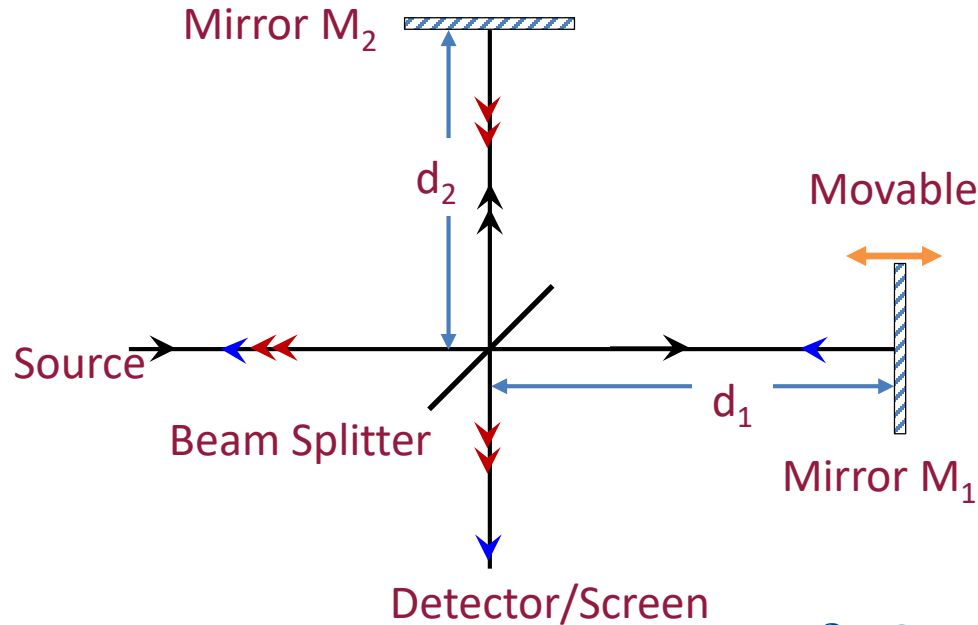


Rainer Weiss, Barry Barish and Kip Thorne

(Nobel Prize, 2017)

130 years old experimental technique!!
Still so useful!!

Basic Design of a Michelson Interferometer



$$\text{Path difference} = 2(d_1 - d_2)$$

$$\text{Phase Difference } (\delta) = \frac{2\pi}{\lambda} \cdot 2(d_1 - d_2)$$

$$\delta = 2m\pi, m = 0, 1, 2, \dots \text{Constructive Interference}$$

$$\delta = (2m + 1)\pi, m = 0, 1, 2, \dots \text{Destructive Interference}$$

LIGO - Laser Interferometer Gravitational Wave Observatory

To detect Gravitational waves, one of the predictions of Einstein's General Theory of Relativity



Hanford Nuclear Reservation,
Washington, Livingston, Louisiana

When Gravitational waves pass through the interferometer they will displace the mirrors!

Arm length: 4 Km

Displacement Sensitivity: 10^{-16} cm

Bright and Dark Fringes in Michelson interferometer

$$\text{Path difference} = 2(d_1 - d_2) = 2d \text{ (say)}$$

$$2d = m\lambda \text{ (m = 0,1,2,...):Maxima}$$

$$2d = \left(m + \frac{1}{2}\right)\lambda \text{ (m = 0,1,2,...):Minima}$$

Order of the fringe:

When the central fringe is bright, the order of the fringe is

$$m = \frac{2d}{\lambda}$$

- As d is increased new fringes appear at the center and the existing fringes move outwards, and finally move out of the field of view.
- For any value of d , the central fringe has the largest value of m .

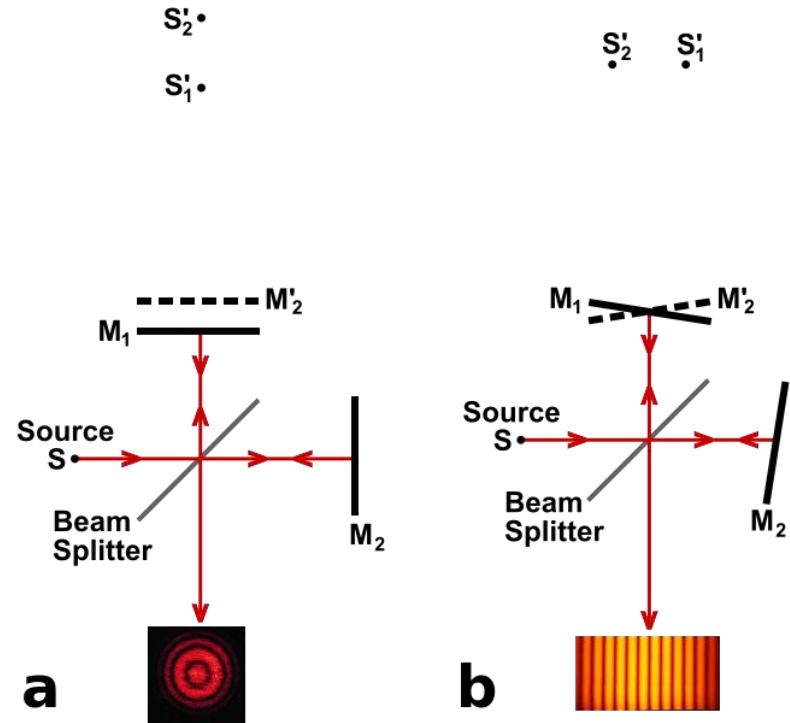




Formation of fringes in Michelson Interferometer

Collimated Source: Uniform illumination

Extended source: Fringe patterns



Applications

1. Measurement of wavelength of light

$$2d\cos\theta = m\lambda$$

$$2d = m_0\lambda \quad (\theta = 0)$$

Move one of the mirrors to a new position d' so that the order of the fringe at the centre is changed from m_0 to m .

$$2d' = m\lambda$$

$$2|d' - d| = |m - m_0|\lambda = n\lambda$$

$$\lambda = \frac{2\Delta d}{\Delta m}$$



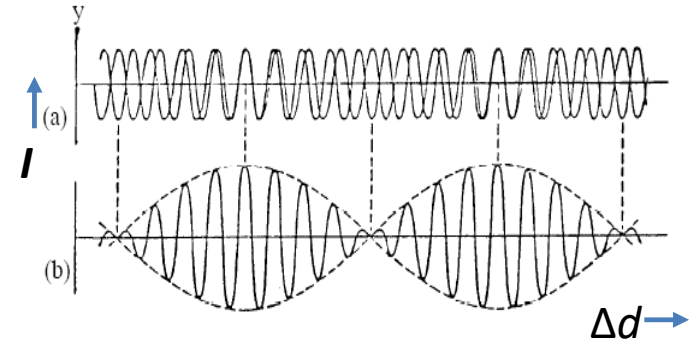
2. Measurement of wavelength separation of a doublet λ_1 and λ_2 ($=\lambda_1+\Delta\lambda$)

Resultant= Interference due to λ_1 + Interference due to λ_2

λ_1 & λ_2 Beating: Stable interference pattern is not observed

$$I_{1t} = 2I_1 \left[1 + \cos\left(\frac{4\pi}{\lambda_1} \cdot \Delta d\right) \right]; \quad I_{2t} = 2I_2 \left[1 + \cos\left(\frac{4\pi}{\lambda_2} \cdot \Delta d\right) \right]$$

$$I = 2(I_1 + I_2) + 2I_1 \cos\left(\frac{4\pi}{\lambda_1} \cdot \Delta d\right) + 2I_2 \cos\left(\frac{4\pi}{\lambda_2} \cdot \Delta d\right)$$



The two λ s beat. We cannot measure it. All we can measure is the Intensity. Also, if λ_1 & λ_2 are quite different, beating will be too fast to be observed.

Suppose we are given the beat curve. We can draw inferences about λ_1 & λ_2 by fitting some value to $(4\pi\Delta d / \lambda)$, since Δd is known.

$$2\Delta d = m\lambda_1 \text{ (For Constructive Interference)}$$

$$2\Delta d = (m + 1/2)\lambda_2 \text{ (For Destructive Interference)}$$

$$\Rightarrow 2\Delta d \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right] = \frac{1}{2}$$

$$\Rightarrow 2\Delta d \left[\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right] = \frac{1}{2}$$

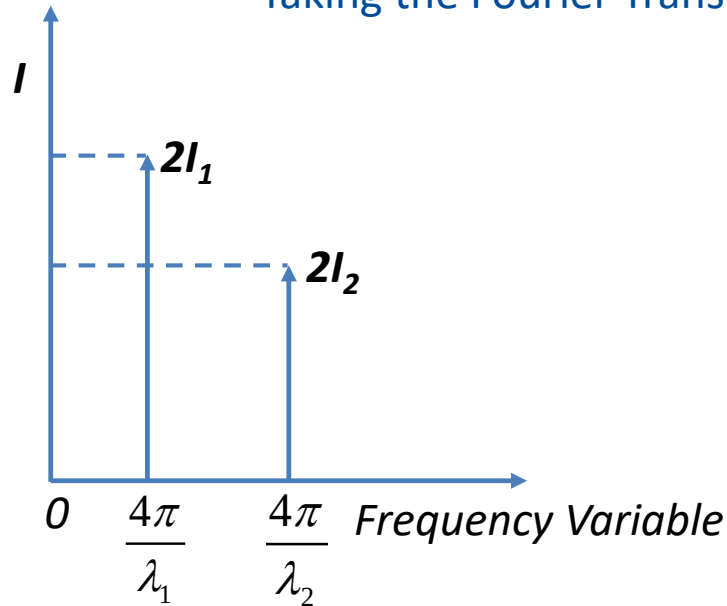
$$\Rightarrow (\lambda_1 - \lambda_2) = \frac{\lambda_1 \lambda_2}{4\Delta d}$$

For very small wavelength difference, $\lambda_1 = \lambda_2 = \lambda$

$$\Rightarrow \Delta\lambda = \frac{\lambda^2}{4\Delta d}$$

$$I = 2(I_1 + I_2) + 2I_1 \cos\left(\frac{4\pi}{\lambda_1} \cdot \Delta d\right) + 2I_2 \cos\left(\frac{4\pi}{\lambda_2} \cdot \Delta d\right)$$

Taking the Fourier Transform of the above Equation :



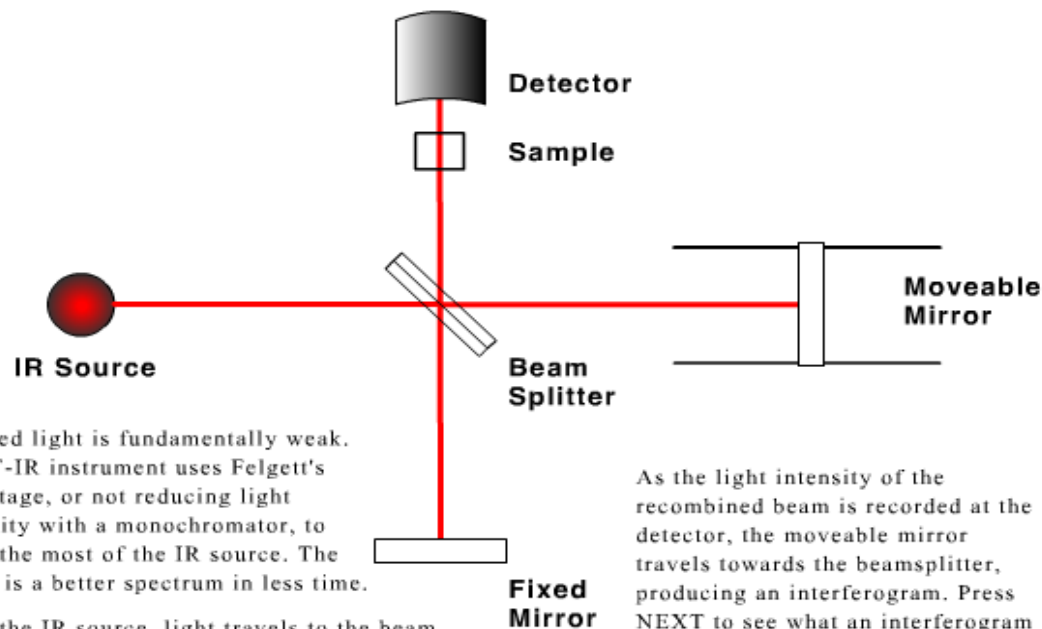
We obtain the spectra, i.e. λ s present in the source. It not only gives λ s, but also the corresponding intensities.

This is called Fourier Transform Spectroscopy.

Determination of Glucose Concentration in Whole Blood using FTIR Spectroscopy



What is FTIR Spectroscopy?



Infrared light is fundamentally weak. An FT-IR instrument uses Fellgett's advantage, or not reducing light intensity with a monochromator, to make the most of the IR source. The result is a better spectrum in less time.

From the IR source, light travels to the beam splitter. 50% of the light is reflected to the fixed mirror and 50% is transmitted to the moveable mirror. Light travels to each of the mirrors and recombines at the beam splitter before passing through the sample and to the detector.

As the light intensity of the recombined beam is recorded at the detector, the moveable mirror travels towards the beamsplitter, producing an interferogram. Press NEXT to see what an interferogram is and how it is constructed.

What is FTIR Spectroscopy?

The reference mirror is scanned and the interferogram is recorded:

$$I(\omega, t) = \int_0^{\infty} S(\omega) T(\omega) \left\{ R_1 + R_2 + 2 \sqrt{R_1 R_2} \cos(\varphi_0 + \omega t) \right\} d\omega$$

This is the cosine Fourier transform of the source spectral power $S(\omega)$ multiplied by the transmission of the sample $T(\omega)$

Hence FT of $I(\omega, t)$ and normalization to the source gives $T(\omega)$

Or the absorption: $A(\omega) = 1 - T(\omega)$

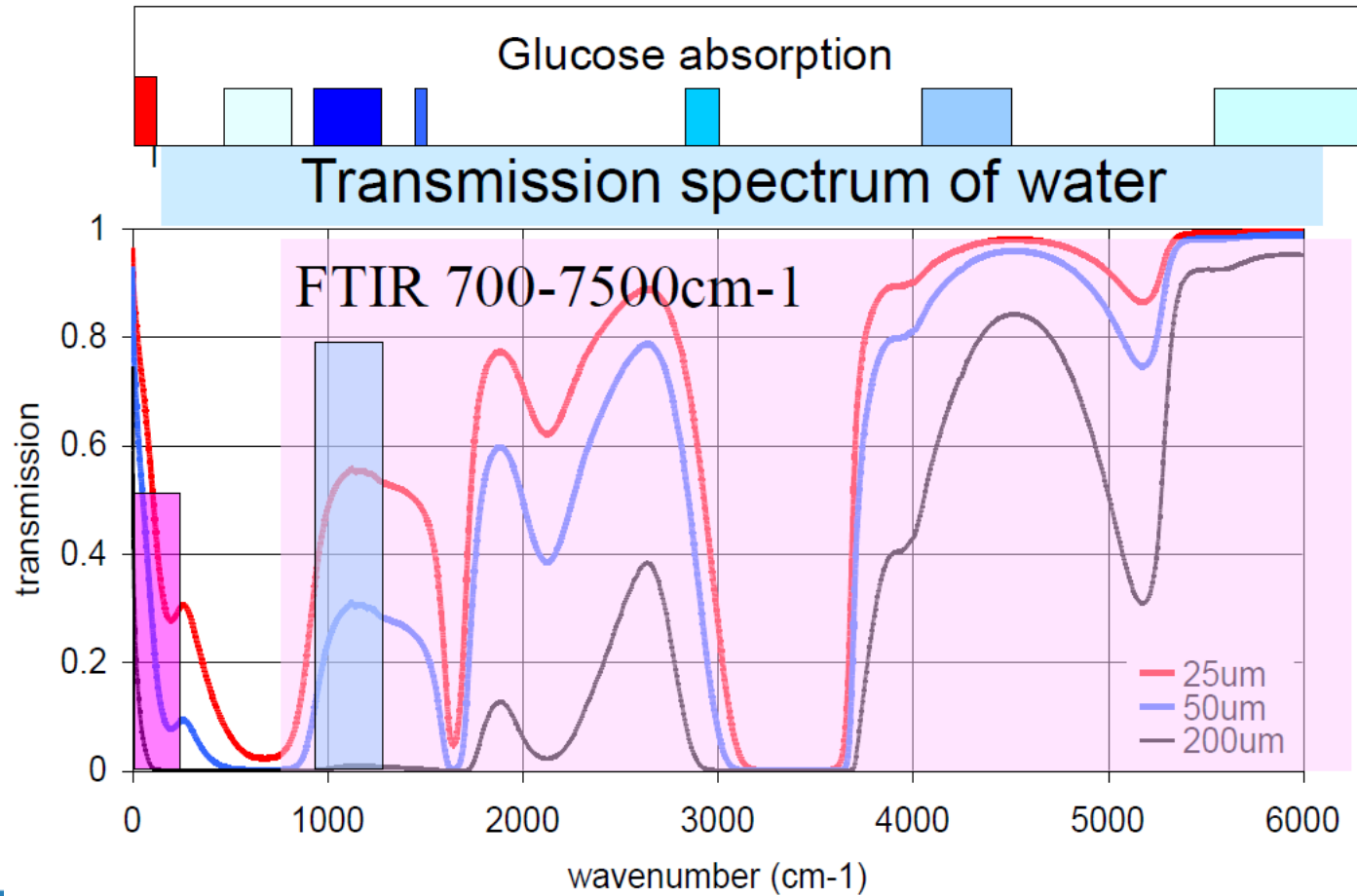
Background

- **Whole blood:**
 - *45% cellular components
 - *55% water and dissolved solids
 - *Clinical level of glucose: 2-30mM (**5mM, 90mg/dL**), (remember: 1mM=18mg/dL)

Normal glucose level

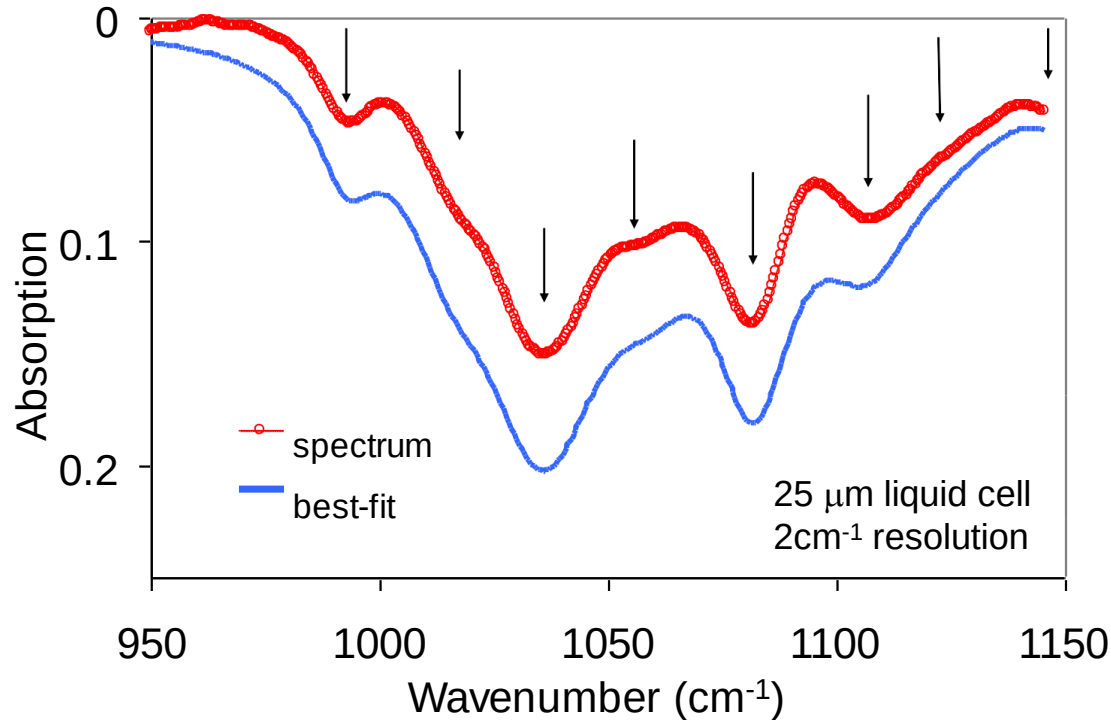
FTIR spectroscopy

- High water absorption
- Complex blood matrix



Absorption spectrum of glucose solution (60mM)

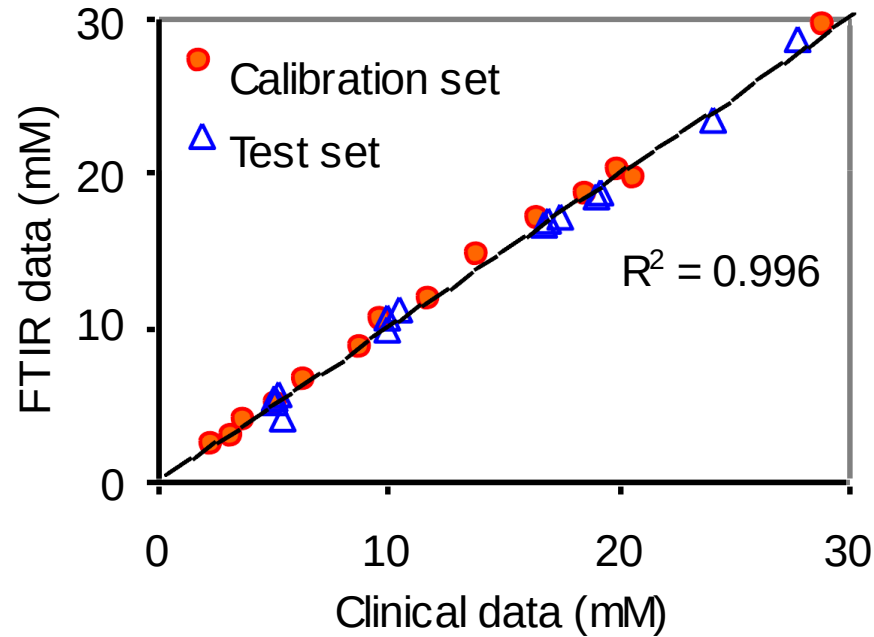
FTIR Spectrum



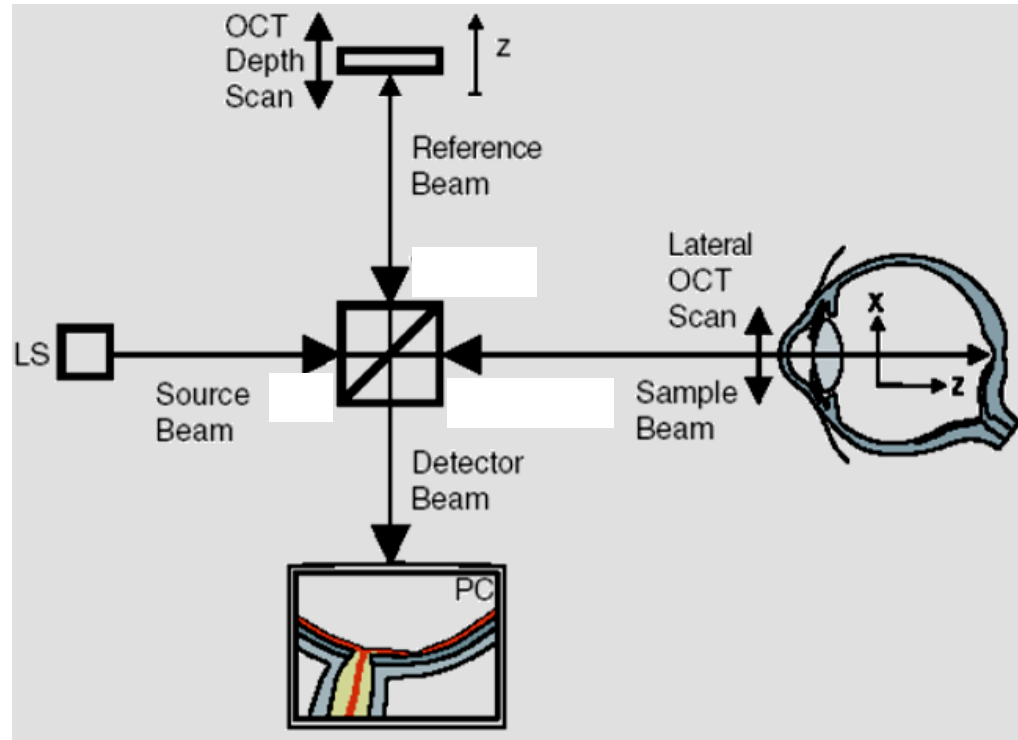
The spectrum can be well simulated using eight Lorentz oscillators

Shen et.al. THz bridge conference, Capri, Italy 2002

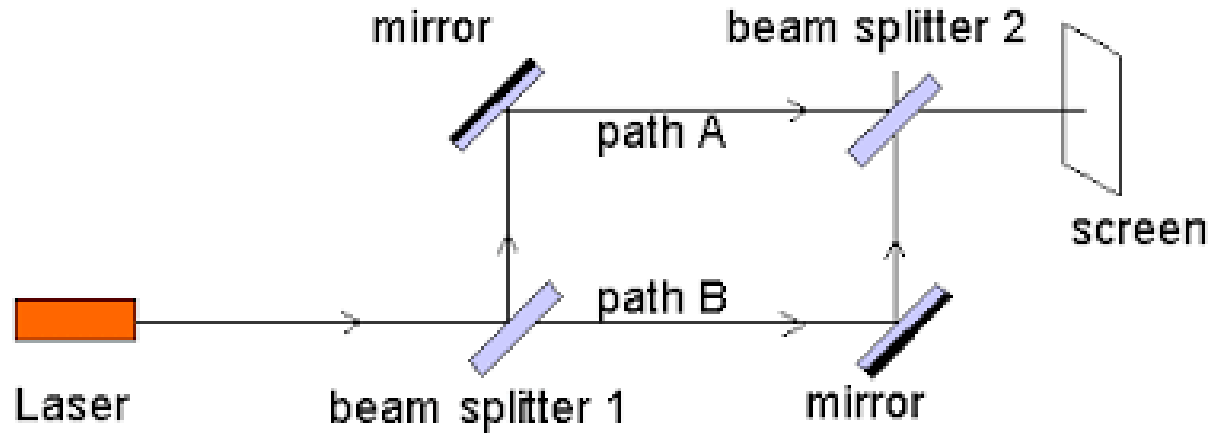
Glucose in whole blood



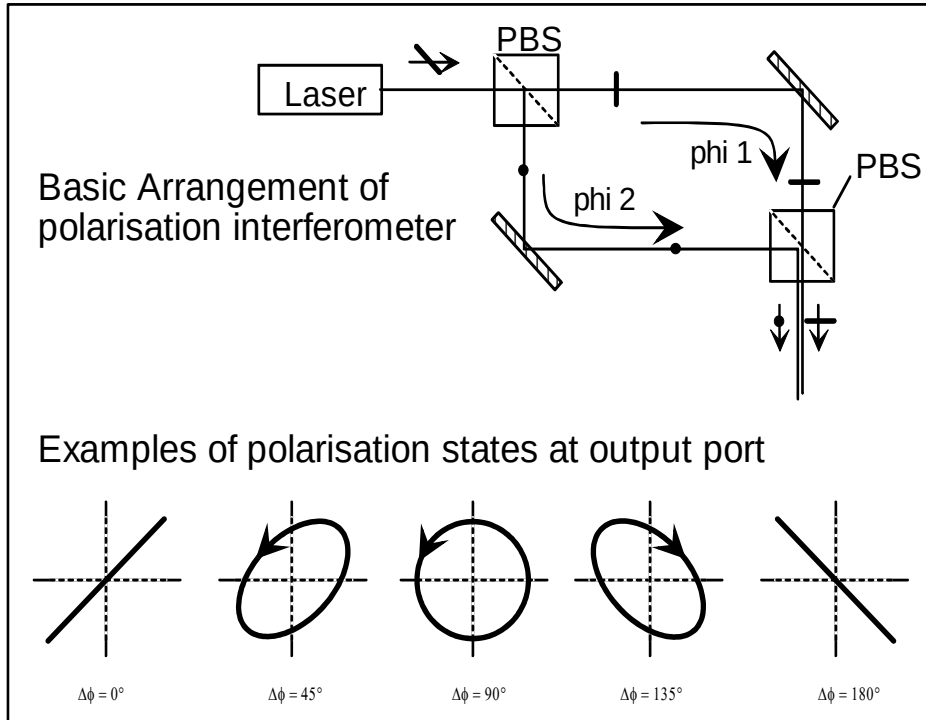
3. Optical Coherence Tomography



Mach-Zehnder Interferometer



Mach-Zehnder interferometer with directional sensitivity



Superposition of 2 orthogonally polarized waves yields not only output intensity but polarization ellipse.

Polarization ellipse carries two informations:

Shape *and* elevation angle.

Or:
Phase difference *and* direction.

Integrated Interferometric Biosensors using Evanescent Waves

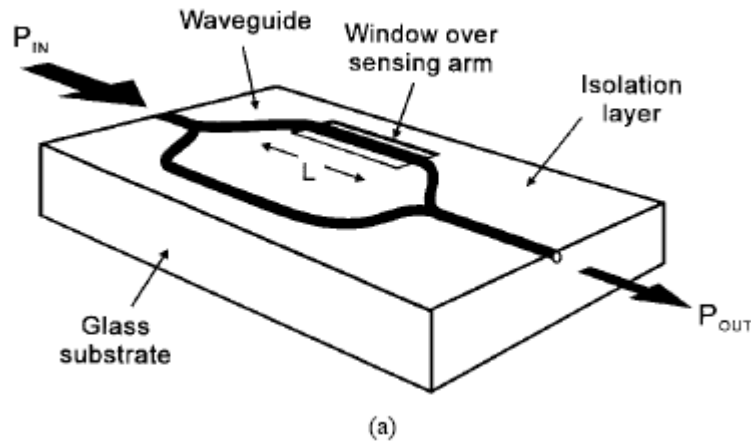
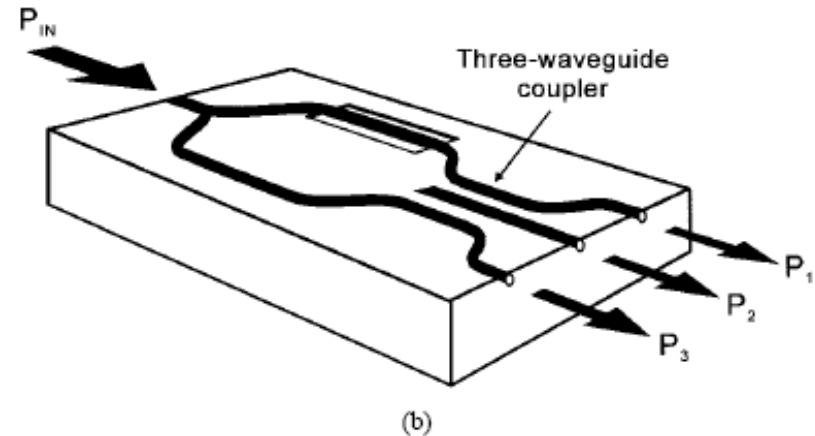


Fig. 1. Mach-Zehnder device configurations.

Luff et.al, J.Lightwave Technolo. 16 (4) 583 (1998)

SPR Interferometer

- The phase change of the reflected p – polarized light is given by the argument of the reflection coefficient:

$$r = |r|e^{j\delta}$$

- For measuring phase change it is possible to use interferometry. The s-polarized light can be used as the reference and p- polarized as the measuring beam.

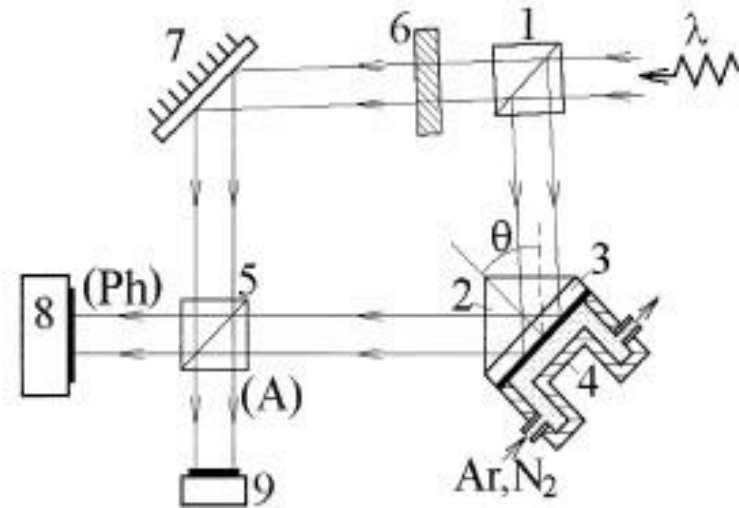


Fig. 1. Scheme of SPR interferometer. (1), (5) beam splitters; (2) prism; (3) slide with Au film; (4) gas cell; (6) absorbing filter; (7) mirror; (8) CCD-matrix; (9) wide-aperture photo diode.

SPR Interferometer: Differential Phase Measurement

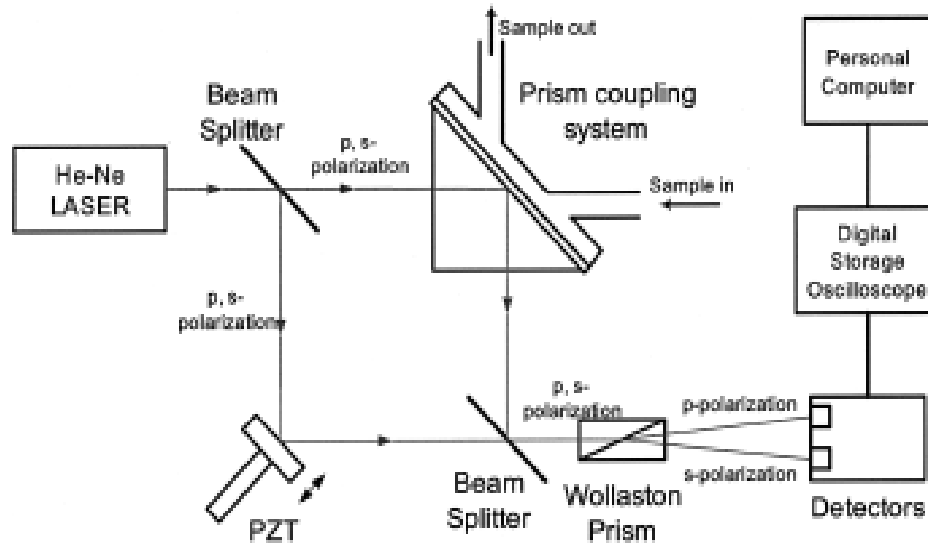


Fig. 1. Experimental setup for measurement of differential SPR phase shift indicating *s* and *p* polarizations on the figure.

- Since the P-phase only is altered by the SPR, this allows to remove the noise.
- Sensitivity is of the order of 5.5×10^{-8} RIU per 0.01 degs of phase change.

Wu et.al., Opt.Lett. 29 (20) 2378 (2004)

Summary

1. Interference and basics of Michelson interferometer were studied in detail
2. Applications of Michelson interferometer were studied rigorously and were used for FTIR based sensing
3. Mach-Zehnder interferometer was discussed in brief and integrated sensors based on it were presented.

Thank You